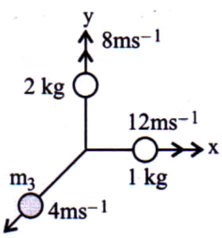
WEEKLY TEST TARGET - JEE -01 TEST - 10
SOLUTION Date 14-07-2019

[PHYSICS]

1.

The situation of the problem is as shown in the figure. According to law of conservation of linear momentum



$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$$

$$\therefore \quad \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$$

Here,

$$\vec{p}_1 = (1\text{kg})(12\text{ ms}^{-1})\hat{i} = 12\,\hat{i}\text{ kg ms}^{-1}$$

$$\vec{p}_2 = (2\text{ kg})(8\text{ ms}^{-1})\hat{j} = 16\hat{j}\text{ kg ms}^{-1}$$

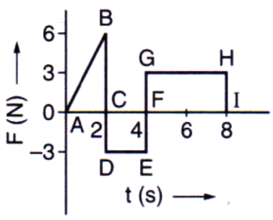
$$\therefore \quad \vec{p}_3 = -(12\,\hat{i} + 16\hat{j})\text{ kg ms}^{-1}$$

The magnitude of $\vec{p}_3$ is :

$$p_3 = \sqrt{(12)^2 + (16)^2} = 20\text{ kg ms}^{-1}$$

$$\therefore \quad m_3 = \frac{p_3}{v_3} = \frac{20\text{ kg ms}^{-1}}{4\text{ ms}^{-1}} = 5\text{kg}$$

2.

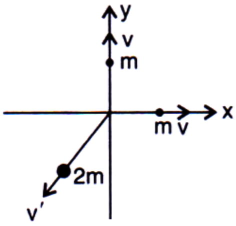


Change in momentum = Area under F-t graph in that interval

= Area of ΔABC – Area of rectangle $CDEF$ + Area of rectangle $FGHI$

$$= \frac{1}{2} \times 2 \times 6 - 3 \times 2 + 4 \times 3 = 12\text{ N s}$$

3.



Let $\vec{v}'$ be velocity of third piece of mass $2m$. Initial momentum, $\vec{P}_i = 0$ (As the body is at rest). Final momentum,

$\vec{P}_f = mv\hat{i} + mv\hat{j} + 2m\vec{v}'$

According to law of conservation of momentum

$\vec{p}_i = \vec{p}_f$

$0 = mv\hat{i} + mv\hat{j} + 2m\vec{v}'$

$\vec{v}' = -\frac{v}{2}\hat{i} - \frac{v}{2}\hat{j}$

The magnitude of $\vec{v}'$ is

$v' = \sqrt{\left(-\frac{v}{2}\right)^2 + \left(-\frac{v}{2}\right)^2} = \frac{v}{\sqrt{2}}$

Total kinetic energy generated due to explosion

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)v'^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)\left(\frac{v}{\sqrt{2}}\right)^2 = mv^2 + \frac{mv^2}{2}$

$= \frac{3}{2}mv^2$

4.

Given that,

$\vec{F} = (2t\hat{i} + 3t^2\hat{j})$ and $\vec{a} = 2t\hat{i} + 3t^2\hat{j}$

Hence, $v = \int_0^t a\,dt = t^2\hat{i} + t^3\hat{j}$

$\therefore \quad P = \vec{F} \cdot \vec{v} = 2t \cdot t^2 + 3t^2 \cdot t^3 = 2t^3 + 3t^5$

5.

Power delivered in time T is,

$$P = F \cdot V = MaV$$

or $$P = MV\frac{dV}{dT}$$

or $$PdT = MVdV$$

or $$PT = \frac{MV^2}{2}$$

or $$P = \frac{1}{2}\frac{MV^2}{T}.$$

6.

Here, $m_1 = m, m_2 = 2m$

$u_1 = 2$ m/s, $u_2 = 0$

Coefficient of restitution, $e = 0.5$

Let v_1 and v_2 be their respective velocities after collision.

Applying the law of conservation of linear momentum, we get,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$\therefore$ $m \times 2 + 2m \times 0 = m \times v_1 + 2m \times v_2$

or $2m = mv_1 + 2mv_2$

or $2 = (v_1 + 2v_2)$ ...(i)

By definition of coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

or $e(u_1 - u_2) = (v_2 - v_1)$

$0.5(2 - 0) = (v_2 - v_1)$

$1 = v_2 - v_1$ ...(ii)

Solving equations (i) and (ii), we get,

$$v_1 = 0 \text{ m/s}, v_2 = 1 \text{ m/s}$$

7.

According to conservation of momentum

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v,$$

where v is common velocity of the two bodies.

$m_1 = 0.1$ kg, $m_2 = 0.4$ kg

$v_1 = 1$ m/s, $v_2 = -0.1$ m/s

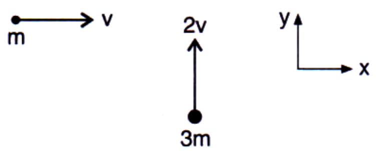
$\therefore$ $0.1 \times 1 + 0.4 \times (-0.1) = (0.1 + 0.4)v$

or $0.1 - 0.04 = 0.5v$,

$$v = \frac{0.06}{0.5} = 0.12 \text{ m/s.}$$

Hence, distance covered = $0.12 \times 10 = 1.2$ m

8.



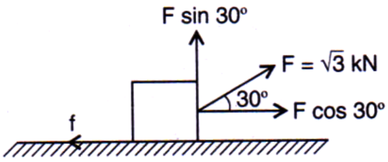
According to conservation of momentum, we get

$$mv\hat{i} + (3m)2v\hat{j} = (m + 3m)v'$$

where v' is the final velocity after collision

$$v' = \frac{1}{4}v\hat{i} + \frac{6}{4}v\hat{j} = \frac{1}{4}v\hat{i} + \frac{3}{2}v\hat{j}.$$

9.



The component of applied force F in the direction of motion is $F\cos 30°$.

The work done by the applied force is,

$$W = (F\cos 30°)S = \sqrt{3}\times 10^3 \times \frac{\sqrt{3}}{2}\times 10\,\text{J}$$

$$= 15\times 10^3\ \text{J} = 15\ \text{kJ}.$$

10.

Mass of water falling/second = 15 kg, $h = 60$ m

$g = 10\,\text{m/s}^2$, loss =10%, *i.e.*, 90% is used

Power generated $= 15\times 10\times 60\times 0.9 = 8100$ W $= 8.1$ kW

11.

$mv = Mv'$ or $v' = \left(\frac{m}{M}\right)v$

Total KE of the bullet and the gun $= \frac{1}{2}mv^2 + \frac{1}{2}Mv'^2$

$$\text{Total KE} = \frac{1}{2}mv^2 + \frac{1}{2}M\cdot\frac{m^2}{M^2}v^2$$

$$\text{Total KE} = \frac{1}{2}mv^2\left[1+\frac{m}{M}\right]$$

or $$1.05\times 1000\ \text{J} = \left[\frac{1}{2}\times 0.2\right]\left[1+\frac{0.2}{4}\right]v^2$$

or $$v^2 = \frac{4\times 1.05\times 1000}{0.1\times 4.2} = (100)^2;$$

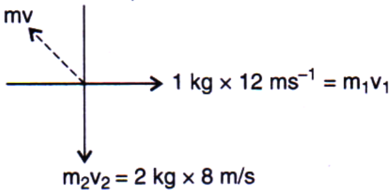
$\therefore$ $$v = 100\ \text{ms}^{-1}$$

12.

When an explosion breaks a rock, by the law of conservation of momentum, initial momentum which is zero, is equal to total momentum of three pieces.

Total momentum of the two pieces 1 kg and 2 kg

$$= \sqrt{12^2 + 16^2} = 20 \text{ kg m s}^{-1}$$



The third piece has the same momentum and in the direction opposite to the resultant of these two momenta.

$\therefore$ Momentum of the third piece $= 20 \text{ kg ms}^{-1}$;

Velocity $= 4 \text{ ms}^{-1}$

$\therefore$Mass of the 3rd piece $= \frac{mv}{v} = \frac{20}{4} = 5$ kg.

13.
14.
15.
16.
17.

Velocity of total mass $(u) = 0$ (because it is stationary). According to law of conservation of momentum

$$(m_1 + m_2)u = m_1v_1 + m_2(-v_2)$$

or $$(m_1 + m_2) \times 0 = m_1v_1 - m_2v_2$$

or $$m_1v_1 = m_2v_2$$

or $$\frac{v_1}{v_2} = \frac{m_2}{m_1}$$

We also know that;

Kinetic energy, $(E) = \frac{1}{2}mv^2 \propto mv^2$

$\therefore$ $$\frac{E_1}{E_2} = \left(\frac{m_1}{m_2}\right) \times \left(\frac{v_1}{v_2}\right)^2$$

$$= \frac{m_1}{m_2} \times \left(\frac{m_2}{m_1}\right)^2 = \frac{m_2}{m_1}.$$

18.

When the mass attached to a spring fixed at the other end is allowed to fall suddenly, it extends the spring by x. Potential energy lost by the mass is gained by the spring,

$$Mgx = \frac{1}{2}kx^2$$

or $$x = \frac{2Mg}{k}.$$

19.

Work done = area under $F - x$ curve

$= $ area of trapezium

$$= \frac{1}{2} \times (6 + 3) \times 3 = 13.5 \text{ J.}$$

20.
21.
22.

Given that $\frac{dW}{dt} = P = K$

or, $W = Pt = \frac{1}{2} mv^2$

or, $\sqrt{\frac{2Pt}{m}} = v$

Hence, $a = \frac{dv}{dt} = \sqrt{\frac{2P}{m}} \frac{1}{2\sqrt{t}}$

Hence, force $= ma = \sqrt{\frac{2Pm^2}{m}} \frac{1}{2\sqrt{t}}$

$$= \left[\sqrt{\frac{mK}{2}}\right] t^{-1/2} \qquad (\because P = K)$$

23.

Because the collision is perfectly inelastic, hence the two blocks stick together. By conservation of linear momentum, $2mV = mv$ or $V = v/2$

By conservation of energy,

$$mgh = \frac{1}{2} mV^2 = \frac{1}{2} m \cdot \frac{v^2}{4} \quad \text{or} \quad h = \frac{v^2}{8g}.$$

24.

$$u_1 = \sqrt{2gh_1}, \quad v_1 = \sqrt{2gh_2}$$

$$e = \frac{v_1 - v_2}{u_2 - u_1}$$

Since, $u_2 = v_2 = 0$,

$$\therefore \quad e = -\frac{v_1}{u_1} = \sqrt{\frac{h_2}{h_1}}.$$

25.

Loss in potential energy $= mgh$

$= 2 \times 10 \times 10 = 200$ J

Gain in kinetic energy = Work done = 300 J

$\therefore$ Work done against friction $= 300 - 200 = 100$ J.

26.
27.

As the force is internal, $\vec{p}_{\text{Th}} + \vec{p}_\alpha = 0$

(as initially system was at rest)

$$(\vec{p}_{\text{Th}})^2 = (-\vec{p}_\alpha)^2 \quad \text{or} \quad p_{\text{Th}}^2 = p_\alpha^2$$

or $K_{\text{Th}} m_{\text{Th}} = K_\alpha m_\alpha$ or $K_{\text{Th}} = \frac{4}{234} \times 4.1$

$= 0.07008$ MeV.

28.

Applying the law of conservation of momentum we get; $mv_0 + 0 = 2m \times v$ or $v = \frac{v_0}{2}$

$$KE = \frac{1}{2}(2m)v^2 = \frac{1}{2} \times 2m \times \left(\frac{v_0}{2}\right)^2 = \frac{mv_0^2}{4}$$

Let the system reach a height h.

Potential energy of the system = $2mgh$

Hence, $\frac{mv_0^2}{4} = 2mgh$ or $h = \frac{v_0^2}{8g}$.

29.

After collision if bullet gets embedded in the block and block rises to a height h, then initial velocity of bullet,

$v = \frac{(M+m)}{m} \cdot \sqrt{2gh}$ (Refer to question 104)

$\therefore$ $v = \sqrt{2 \times 980 \times 2.5}\left(\frac{5010}{10}\right) = 350.7$ m/sec.

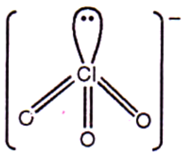
30.

Power $P = Fv = \frac{K}{v} \cdot v = K$ = constant

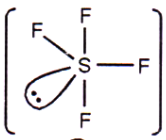
$\therefore$ $W = Pt = Kt.$

[CHEMISTRY]

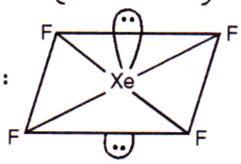
31. Structures of the compounds are as follows:

$[ClO_3]^-$: 

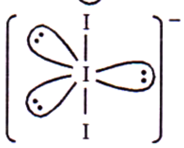
(One lone pair)

SF_4 : 

(One lone pair)

XeF_4 : 

(Two lone pairs)

$[I_3]^-$: 

(Three lone pairs)

32. Electronegativity of carbon increases with increase in s-character of hybrid orbitals.

33.

34.